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ON THE IMPACT OF THE TESTS FOR SERIAL CORRELATION UPON THE TEST OF SIGNIFICANCE FOR THE REGRESSION COEFFICIENT*

Alice NAKAMURA and Masao NAKAMURA

Faculty of Business Administration and Commerce, University of Alberta, Edmonton, Alberta, Canada

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Monte Carlo methods are used to investigate the relationship between the power of different pretests for autocorrelation, and the Type I error and power of the significance test for a resulting two-stage estimate of the slope parameter in a simple regression. Our results suggest it may be preferable to always transform without pretesting. Moreover we find little room for improvement in the Type I errors and power of two-stage estimators using existing pretests for autocorrelation, compared with the results obtained given perfect knowledge about *when* to transform (i.e., given a perfect pretest). Rather, researchers should seek better estimators of the transformation parameter itself.

1. Introduction

There is a vast literature concerned with the implications of autoregressive disturbances in a linear regression model, and with various alternative estimation methods which may yield better results for such a model. [See Cochrane and Orcutt (1949), Durbin (1960), Fomby and Guilkey (forthcoming), Hildreth and Lu (1960), and Sargan (1964).] Of course, if we do not know whether the model we are dealing with has an autoregressive disturbance term, we may not be able to decide which estimation method to use. This problem has led to interest in the relative power and other properties of different tests of the null hypothesis of no autocorrelation. [See Durbin and Watson (1950, 1951), Jenkins (1954, 1956), Hannan (1957), McGregor (1960), Theil and Nagar (1961), Henshaw (1966), Hannan and Terrell (1968), Koerts and Abrahamse (1968), Abrahamse and Koerts (1967), Geary (1970), Habibagahi and Pratschke (1972), Belsley (1973), Blattberg (1973), Harrison (1975), and Schmidt and Guilkey (1975).] However, relatively little attention has been paid to the impact of the procedure used to test the hypothesis of no autocorrelation on the final results achieved in estimating the regression coefficients of the assumed linear relationship.

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Kmenta (1971, p. 296) observes that 'in most studies concerned with estimating regression equations from time-series data, the value of the d statistic is presented along with the other estimates. The question then is what action, if any, is to be taken in response to a particular outcome of the test? If no autoregression is indicated, we can retain the least squares estimates...' When autoregression is indicated, an autoregressive transformation of some sort is often performed. What if some alternative test for autocorrelation is used though? Habibagahi and Pratschke (1972) report the results of a Monte Carlo experiment designed to compare the powers of the Durbin–Watson and Geary tests for autocorrelation. They find that in the limiting case of no autocorrelation the concordance of the Durbin–Watson and Geary tests is not close, despite the fact that both tests appear to yield approximately the same Type I error. Harrison (1975), and Schmidt and Guilky (1975) have replicated the Monte Carlo experiment carried out by Habibagahi and Pratschke, and have presented evidence questioning some of their conclusions. Nevertheless Habibagahi and Pratschke's work still brings to mind an interesting set of questions.

A different set of decisions concerning the presence of autocorrelation would result in a different set of decisions concerning when to retain the original least squares regression results and when to adopt some other estimation procedure. How much difference would this make in terms of the final estimation results? Is it crucial to continue the search for new tests for autocorrelation? How does this problem rank in importance relative to other problems encountered in the estimation of autoregressive relationships? We will attempt to address these questions in the context of the following model.

2. The model

We have chosen to use the model

$$y_t = \alpha+\beta t+\varepsilon_t, \qquad t = 1, 2, \ldots, T, \tag{1}$$

and

$$\varepsilon_t = \rho\varepsilon_{t-1}+\mu_t,$$

where the μ_t are independently, normally distributed with mean 0 and variance 1. The explanatory variable in this model can be thought of as a first-order autoregressive process with an autoregressive parameter equal to 1.0 and a random error term identically equal to zero. This is the model which was used by Habibagahi and Pratschke (1972), Harrison (1975) and Schmidt and Guilkey (1975) to compare the powers of the Durbin–Watson and Geary tests for serial correlation.

We have carried out our Monte Carlo experiments[1] for $T = 30, 50$, $\rho = 0.0$, 0.3, 0.5, 0.7, 0.9, and $\beta = 0.0, 1.5$, using generated samples of 1,000 repetitions

[1]We used in our study the Chen random normal number generator [Chen (1971)] for which satisfactory statistical properties are reported. The computer used is an Amdahl 470V/6 at the University of Alberta computer center.

for each combination of values of T, ρ, and β. For each repetition ordinary least squares was used to estimate model (1), and the least squares residuals were calculated.

Six different decision rules were then used to test the null hypothesis $H_0:\rho = 0$ against the one-sided alternative $H_2:\rho > 0$ in our Monte Carlo environment where $\rho \geqq 0.0$:

(1) No test; always assume $H_0:\rho = 0$.

(2) Geary test using the lower critical value at a 5 percent level of significance.[2]

(3) Durbin–Watson test using the lower bound at a 5 percent level of significance.

(4) Joint Geary and Durbin–Watson test using the lower critical value for the Geary test and the lower bound for the Durbin–Watson test at a 5 percent level of significance for each of the two tests.

(5) Durbin–Watson test using the upper bound at a 5 percent level of significance.[3]

(6) No test; always assume $H_a:\rho > 0$.

For each pair of y and t series, if $H_0:\rho = 0$ is accepted then a t-statistic is used to test the null hypothesis $H_0:\beta = 0$ against the two-sided alternative hypothesis $H_a:\beta \neq 0$ at a 5 percent level of significance. If $H_0:\rho = 0$ is rejected in favor of the alternative $H_a:\rho > 0$, then the Cochrane–Orcutt iterative technique is used to reestimate model (1). The results from this transformed regression are then used to test the hypothesis $H_0:\beta = 0$ against the alternative $H_a:\beta \neq 0$ at a 5 percent level of significance.[4]

[2]For $T = 30$, $P(\tau \leqq 9) = 0.0307$ while $P(\tau \leqq 10) = 0.0680$. For $T = 50$, $P(\tau \leqq 18) = 0.0427$ while $P(\tau \leqq 19) = 0.0762$, where τ is the number of sign changes in the residuals. [See Geary (1970) and Habibagahi and Pratschke (1972).] Schmidt and Guilkey (1975) suggest that this indeterminancy should be resolved by the use of a randomized rejection scheme when the value of τ falls between the lower and upper critical values of τ. While this scheme does result in the appropriate Type I error in a Monte Carlo experiment, we doubt whether a practitioner testing for autocorrelation in a single series of regression residuals would employ such a scheme.

[3]Theil and Nagar (1961) showed that the upper bound for the Durbin–Watson test is approximately equal to the true significance limit when the behavior of the explanatory variable is smooth in the sense that the first and second differences are small compared with the range of the explanatory variable itself. This is, in fact, the case for the explanatory variable in model (1). Schmidt and Guilkey (1975) criticized Habibagahi and Pratschke (1972) for using the lower bound for the Durbin–Watson test on these grounds. Schmidt and Guilkey's Monte Carlo results support their criticism.

[4]The Cochrane–Orcutt iterative transformation used in this study is identical to the version of the Cochrane–Orcutt iterative technique available as an option of the Time Series Processor (TSP) regression package. For a description of this transformation in TSP, see, for example, Hoskin (1974, pp. 32–33). See also Kmenta (1971, p. 288). Iteration was terminated when the change in the estimate of ρ was less than 0.005, or 20 iterations had occurred. The iterative process converged in considerably less than 20 iterations for all cases.

3. Our results when var $(\mu_t) = 1$

In table 1 we show the powers of our tests of significance for ρ using decision rules 2–5 for $T = 30$, 50 and $\rho = 0.0, 0.3, 0.5, 0.7, 0.9$. These power results are not affected by the value of β. [See Nakamura and Nakamura (1976) for a more complete discussion of this point.] We note that our figures in table 1 are quite similar to those derived by Schmidt and Guilkey (1975). When the lower bound is used for the Durbin–Watson test with a 5 percent level of significance, the true Type I error is actually less than 5 percent. Likewise when the upper bound is used the true Type I error is actually greater than 5 percent. Thus for the case when $\rho = 0.0$, for instance, when the lower bounds are used for $T = 30$ and $T = 50$ respectively the observed Type I errors are 2.4 and 2.6 percent. When the upper bounds are used the corresponding Type I errors are 5.3 and 5.1 percent.

The overall estimation results, in terms of the Type I errors for the test of significance of the null hypothesis $H_0:\beta = 0$, are shown in table 2 for decision rules 1–6 and $T = 30$, 50. For $0.0 < \rho < 1.0$ the Type 1 error is seen to be smaller the larger the number of transformations per sample of 1,000 regressions indicated by the decision rule adopted. The Type I error made for decision rule 6 when we always assume $H_a:\rho > 0$, and hence always transform, is closest to the 5 percent critical region specified for all values of ρ except $\rho = 0.0$. Nevertheless even when we always perform a transformation before testing the significance of β, for $\rho = 0.9$ the Type I error for our test of $H_0:\beta = 0$ is 35.3 percent when $T = 30$ and 28.2 percent when $T = 50$. [Notice that the observed Type I errors when $\rho = 0.0$ and we never transform are 4.4 and 4.3 percent for $T = 30$ and $T = 50$, respectively. Further research has shown that this is due to the high level of autocorrelation of the explanatory variable t and the fact that, even when $\rho = 0.0$, the sample autocorrelations for the disturbance term are not exactly zero. As the autocorrelation parameter of the explanatory variable is increased from 0.0 to 1.0 with $\rho = 0.0$, the observed Type I errors first rise above the specified level of 5% and then ultimately fall below 5 percent. For a preliminary discussion of this question see Nakamura and Nakamura (1973) and Nakamura, Nakamura and Orcutt (1976).]

Due to the small variance of μ in model (1), the corresponding powers for our six decision rules when $\beta = 1.5$ are reassuring, though not very useful for comparative purposes. The power is 100 percent for all cases for $T = 30$ and $T = 50$.

4. Our results when var $(\mu_t) = 625$

In order to be able to observe the differential impact of our six decision rules on the power of the test of significance for β when $\beta = 1.5$, we now modified model (1) by setting the variance of μ at 625. The Monte Carlo experiment described in section 2 was then repeated once again using this modified model.

Table 1

Powers of Geary and Durbin–Watson tests at a 5 percent level of significance.

	$T = 30$				$T = 50$			
ρ	G[a]	D–WL[b]	G&D–W[c]	D–WU[d]	G	D–WL	G&D–W	D–WU
0.0	1.3[e] (2.2, 2.4)[f]	2.4 (2.6, 1.6)	3.4	5.3 (5.7, 4.1)	2.4 (1.6)	2.6 (2.3)	4.1	5.1 (4.9)
0.3	12.8 (13.0, 18.0)	29.2 (24.6, 38.5)	31.8	45.0 (38.9, 50.5)	26.3 (24.2)	53.2 (50.7)	57.1	64.7 (63.7)
0.5	29.4 (45.6, 60.6)	64.0 (61.0, 81.1)	65.4	78.3 (72.7, 88.8)	62.2 (58.8)	91.6 (92.4)	92.3	94.8 (95.9)
0.7	58.7 (55.6, 75.8)	89.7 (84.8, 97.2)	90.7	93.4 (91.2, 98.3)	89.6 (86.3)	99.3 (99.1)	99.4	99.6 (99.8)
0.9	78.1 (73.2, 91.6)	96.1 (94.6, 99.4)	96.3	97.7 (94.6, 99.4)	98.3 (97.6)	99.9 (100.0)	100.0	100.0 (100.0)

[a] G ≡ Geary test with lower critical point.
[b] D–WL ≡ Durbin–Watson test using lower bound.
[c] G&D–W ≡ Joint Geary and Durbin–Watson test.
[d] D–WU ≡ Durbin–Watson test using upper bound.
[e] Each of the numbers in the table gives the percentage of cases for which $H_0: \rho = 0$ was rejected.
[f] The numbers in parentheses below our results for $T = 30$ and $T = 50$ are the comparable results reported by Schmidt and Guilkey (1975, p. 381, table 1) for $T = 27$, 39 and $T = 51$, respectively.

Table 2

Type I errors for test of significance for slope coefficient at 5 percent level of significance, $\beta = 0.0$.

Decision rule[a]	$T=30$					$T=50$				
	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$
1	4.4[b]	14.1	24.5	40.5	62.8	4.3	13.4	26.0	43.4	65.3
2	4.4	13.1	21.0	27.7	39.9	4.2	11.2	15.1	16.0	28.5
3	4.4	12.3	15.4	21.3	35.5	4.2	8.4	10.6	13.5	28.2
4	4.4	12.0	15.4	20.9	35.5	4.2	8.6	10.5	13.5	28.2
5	4.5	11.0	14.0	20.3	35.5	4.2	8.0	10.0	13.5	28.2
6	7.6	9.5	13.2	19.7	35.3	6.2	7.0	10.0	13.5	28.2

[a]See section 2 of the text for the definitions of these six decision rules.
[b]Each of the numbers in the table gives the percentage of cases for which $H_0:\beta = 0$ was rejected.

Table 3

Powers of test of significance for slope coefficient at 5 percent level of significance, $\beta = 1.5$.

Decision rule[a]	$T=30$					$T=50$				
	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$
1	77.7[b]	69.1	65.1	59.0	66.4	100.0	99.9	98.2	87.2	75.7
2	77.2	65.1	53.7	38.8	44.3	100.0	98.7	87.6	60.3	39.6
3	76.1	59.0	44.7	30.7	40.5	100.0	97.6	85.3	58.3	39.4
4	75.9	58.5	44.3	30.7	40.5	100.0	97.6	85.3	58.3	39.4
5	75.5	54.5	42.1	30.3	40.3	100.0	97.5	85.2	58.3	39.4
6	74.0	50.6	40.7	30.1	40.2	100.0	97.5	85.2	58.3	39.4

[a]See section 2 of the text for the definitions of these six decision rules.
[b]Each of the numbers in the table gives the percentage of cases for which $H_0:\beta = 0$ was rejected.

The powers of decision rules 2–5 are the same as those shown in table 1 for the null hypothesis $H_0:\rho = 0$ and the alternative hypothesis $H_a:\rho > 0$, since the Durbin–Watson statistic is invariant with respect to changes in var (μ_t). [See Nakamura and Nakamura (1976).] Also the Type I errors for our test of significance of the null hypothesis $H_0:\beta = 0$ are identical to those shown in table 2 since the Type I errors for model (1) are also invariant with respect to changes in var (μ_t). [See Nakamura and Nakamura (1976).] However, the associated powers shown in table 3 for the test of significance for β when $\beta = 1.5$ are much lower than the uniform 100% observed for all cases for model (1) with var $(\mu_t) = 1$. For instance, when we always perform a transformation prior to testing the significance of β, for $\rho = 0.9$ the Type I errors for this test are 35.3 percent for $T = 30$ and 28.2 percent for $T = 50$ as before, but now the associated powers when $\beta = 1.5$ are only 40.2 percent and 39.4 percent.

The severe drop in the power of our test of significance for β when $\beta = 1.5$ and the variance of μ is increased from 1 to 625 becomes more understandable if we examine the corresponding population correlations between y and t in model (1) for $T = 30$, 50 and $\rho = 0.0, 0.3, 0.5, 0.7, 0.9$. These correlations are shown in table 4. Since the population correlation between y and t is given by

Table 4

Population correlations[a] (ρ_{yt}) between y and t in model (1) when $\beta = 1.5$.

	$T = 30$		$T = 50$	
ρ	var(μ) = 1	var(μ) = 625	var(μ) = 1	var(μ) = 625
0.0	0.9970	0.4609	0.9989	0.6546
0.3	0.9968	0.4439	0.9988	0.6368
0.5	0.9961	0.4102	0.9986	0.5999
0.7	0.9942	0.3477	0.9979	0.5259
0.9	0.9847	0.2207	0.9944	0.3530

[a] $\rho_{yt} = \sqrt{\dfrac{\beta^2\sigma_t{}^2/\sigma_\varepsilon{}^2}{1+(\beta^2\sigma_t{}^2/\sigma_\varepsilon{}^2)}}$. [See Nakamura and Nakamura (1976).]

$\rho_{ty} = [(\beta^2\sigma_t^2/\sigma_\varepsilon^2)/(1+(\beta^2\sigma_t^2/\sigma_\varepsilon^2))]^{\frac{1}{2}}$ where $\sigma_\varepsilon^2 = \text{var}(\mu_t)/(1-\rho^2)$ and σ_t^2 is the variance of the t series. it is seen that, for a given value of $\beta > 0$, the linear relationship between y and t becomes weaker the smaller the value of T, the larger the variance of μ, and the larger the value of ρ. [See Nakamura and Nakamura (1976).]

The pattern of Type I versus Type II errors implied by the power results shown in table 3 deserves some explanation too. For any given model – that is, for any given values of T, β, ρ and var (μ_t) – the number of Type II errors should increase as the number of Type I errors decreases. This is the behavior observed for the Type II errors implied by the power results shown in table 3 and the corresponding Type I errors shown in table 2.

For any given decision rule, however, the number of Type I errors increases as ρ increases while the number of Type II errors increases too, and then begins to fall for T sufficiently small and ρ sufficiently large. This can be explained as follows.

The Type II error for our test of significance of β when $\beta = 1.5$ and we use decision rule 1 (i.e., we always assume $H_0\!:\rho = 0$) is

$$P(-t_{\alpha/2,T-2} < b/S_b < t_{\alpha/2,T-2}), \qquad (2)$$

where b/S_b is assumed to obey a t-distribution with $T-2$ degrees of freedom. Generalizing the relationship presented by Goldberger (1964, p. 242) for the expected value of the usual least squares formula for S_b, we find that $E(S_b^2) = B\sigma_b^2$, where

$$B = \frac{1-2\sum_{t=1}^{T-1}\rho^t\left[\left(\frac{T-t}{T}\right)+\left[\frac{\sum_{i=t+1}^{T} x_i x_{i-t}}{\sum_{t=1}^{T} x_t^2}\right]\right]\Bigg/(T-2),}{1+2\sum_{t=1}^{T-1}\rho^t\left[\frac{\sum_{i=t+1}^{T} x_i x_{i-t}}{\sum_{t=1}^{T} x_t^2}\right]},$$

$$x_t = t-(\textstyle\sum t/T),$$

$$\sigma_b^2 = \sigma_\varepsilon^2\left[\frac{1+2\sum_{t=1}^{T-1}\rho^t\left[\frac{\sum_{i=t+1}^{T} x_i x_{i-t}}{\sum_{t=1}^{T} x_t^2}\right]}{\sum_{t=1}^{T} x_t^2}\right],$$

and b is the least squares estimator of β. [For the derivation of these generalized expressions see Nakamura and Nakamura (1976).] Thus we can write the expected value of the usual formula for the variance of b as the product of a bias term, which we will denote by B, and the true variance of b denoted by σ_b^2.

Substituting $\sqrt{E(S_b^2)}$ for S_b, expression (2) can now be roughly approximated by[5]

$$P(-t_{\alpha/2,T-2} < b/(\sigma_b\sqrt{B}) < t_{\alpha/2,T-2})$$
$$= P\left(-t_{\alpha/2,T-2}\sqrt{B}-\frac{1.5}{\sigma_b} < \frac{b-1.5}{\sigma_b} < t_{\alpha/2,T-2}\sqrt{B}-\frac{1.5}{\sigma_b}\right), \quad (3)$$

where

$$(b-1.5)/\sigma_b \sim N(0, 1).$$

Using expression (3), the approximate values of the power of our test of significance for β for decision rule 1 when $T = 30$ are 78.8, 74.5, 65.9, 50.0 and 63.2 percent for $\rho = 0.0$, 0.3, 0.5, 0.7 and 0.9, respectively. These approximate values for the power follow the same pattern as ρ increases from 0 to 1 as the

[5]Note that $E(S_b) \neq \sqrt{E(S_b{}^2)}$. [It can be shown that $E(S_b) = \sqrt{(E(S_b{}^2)-2(T-2))}$.] The substitution in the text is used only to allow us to interpret the general pattern of falling and then increasing power shown in table 3.

observed values shown in table 3. Thus the pattern of falling and then increasing power shown in table 3 is seen to be due to the opposing effects of decreases in B, the bias term, and increases in σ_b^2, the true variance of the least squares estimator of β, as ρ increases from 0.0 to 0.9.

6. Possible gains from improved transformation schemes

Our results so far suggest that tests of significance for autocorrelation might best be dispensed with in estimating relationships similar to model (1) in favor of a practice of always transforming prior to testing the significance of β. (In other words, our results suggest that decision rule 6 should be substituted for decision rules 2–5.) This is an important point, given the established role which tests for autocorrelation have come to play in applied work, and the considerable effort which theoreticians and others have devoted in recent years to finding new and improved tests for autocorrelation. Moreover for true values of $\rho > 0$, when we always transform without first testing for autocorrelation the final estimation results obtained are what would be achieved given a 'perfect' test for autocorrelation. If these 'perfect' results are only slightly different (slightly better when $\rho > 0$ and slightly worse when $\rho = 0$) from the final results achieved using less perfect tests for autocorrelation such as the Durbin–Watson test, then further effort devoted to improving tests for autocorrelation probably will not result in any significant improvements in applied statistical practices and results. Looking at the bottom two lines of table 2 we see that the Type I errors obtained by transforming only when the Durbin–Watson test using the upper bound indicates significant autocorrelation (decision rule 5) and the Type I errors obtained by always transforming (decision rule 6) differ by at most 3.1 percent for $T = 30$ and by at most 2 percent for $T = 50$.

However, even when we always transform we are still left with unsatisfactorily large Type I errors for the standard test of significance for β when ρ is large. It has been suggested that for time series of modest length obeying a relationship such as model (1) the Durbin modification of the Cochrane–Orcutt iterative technique may improve the efficiency of the resulting estimator of ρ. [See Durbin (1960) and Malinvaud (1966, p. 433).] A variety of other schemes have also been proposed for estimating, or correcting the estimate of ρ. [See Quenouille (1949), Hurwicz (1950), Kendall (1954), Marriott and Pope (1954), and Orcutt and Winokur (1969)]. It is interesting, therefore, to investigate how much improvement we would observe in the Type I errors for our test of significance for β if the true values of ρ could be determined exactly.

In order to investigate this question, we repeated the Monte Carlo experiment described in section 2 once more with the following modifications. The variance of μ was again set equal to 625. Secondly all transformations indicated by our decision rules 1–6 were now performed using the true values of ρ. The results of this third experiment are shown in tables 5 and 6.

Table 5

Powers of test of significance for slope coefficient at 5 percent level of significance with true ρ known, $T = 30$.

Decision rule[a]	$\beta=0.0$					$\beta=1.5$				
	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$
1	4.4[b]	14.1	24.5	40.5	62.8	77.7	69.1	65.1	59.0	66.4
2	4.4	13.0	20.9	22.6	21.2	77.6	65.9	54.1	32.4	24.7
3	4.4	12.7	13.8	11.2	9.6	77.3	61.6	41.7	17.7	11.5
4	4.4	12.3	13.7	10.5	9.4	77.3	60.9	40.9	16.9	11.3
5	4.5	11.3	10.7	9.0	8.3	77.1	56.9	35.2	15.4	10.0
6	4.4	4.9	5.3	5.4	7.0	71.3	41.3	25.7	11.6	7.9

[a]See section 2 of the text for the definitions of these six decision rules.
[b]Each of the numbers in the table gives the percentage of cases for which $H_0: \beta = 0$ was rejected.

Table 6

Powers of test of significance for slope coefficient at 5 percent level of significance with true ρ known, $T = 50$.

Decision rule[a]	$\beta=0.0$					$\beta=1.5$				
	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$	$\rho=0.0$	$\rho=0.3$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$
1	4.3[b]	13.4	26.0	43.4	65.3	100.0	99.9	98.2	87.2	75.7
2	4.3	11.4	13.4	9.5	6.6	100.0	99.5	88.5	48.3	10.3
3	4.3	9.1	7.0	5.4	5.5	100.0	98.5	84.5	42.6	9.0
4	4.3	9.0	6.9	5.3	5.4	100.0	98.5	84.4	42.6	8.9
5	4.4	8.5	6.0	5.3	5.4	100.0	98.4	84.1	42.6	8.9
6	4.3	5.2	4.8	5.1	5.4	100.0	98.1	83.4	42.4	8.9

[a]See section 2 of the text for the definitions of these six decision rules.
[b]Each of the numbers in the table gives the percentage of cases for which $H_0: \beta = 0$ was rejected.

The results shown in these tables indicate that the problem of overly large Type 1 errors for the test of significance of β in relationships similar to model (1) could be virtually eliminated by always performing a transformation prior to testing the significance of β if ρ could be estimated with sufficient accuracy.

These results suggest that more attention should be paid to the development and use of alternatives to the Cochrane–Orcutt iterative technique for estimating autoregressive relationships. More attention should also probably be focused on the possibility of developing tests of significance for autoregressive models which can be applied both with and without transformation of the original data. [See Orcutt and James (1948), Ogawara (1951), Hannan (1955), McGregor (1962), McGregor and Bielenstein (1965), Jenkins and Watts (1968), and Nakamura, Nakamura and Orcutt (1976).

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